AM and high-harmonic FM laser mode locking

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We demonstrate a new technique of active mode locking that combines amplitude-modulated (AM) mode locking at the cavity fundamental repetition rate with frequency-modulated (FM) mode locking at a high harmonic. This method combines the advantages of pulse shortening by high-harmonic mode locking while preserving the higher peak powers available at the fundamental repetition rate. We demonstrate this technique using a Nd:YAG laser that is simultaneously AM mode locked at 80 MHz and FM mode locked at the 22nd harmonic (1.76 GHz). Pulses as short as 16 ps with a peak power of 6.25 kW were measured. © 1997 Optical Society of America

Key words: Active mode locking, mode-locked laser, simultaneous mode locking, amplitude-modulated mode locking, frequency-modulated mode locking, harmonic mode locking.

1. Introduction

Modern approaches to ultrashort laser-pulse generation have been dramatically successful by relying on nonlinear effects such as Kerr lens mode locking[1] in Ti:sapphire, soliton pulse shaping in erbium-doped fiber lasers,[2,3] and saturation effects in semiconductors.[4] Early approaches to laser mode locking such as intracavity amplitude modulation or frequency modulation suffered from a requirement of high drive signals and/or frequencies to exploit the full gain bandwidth available in the host laser medium. By today's standards, active mode locking is not competitive with passive mode locking, yet there are still a few applications in which active mode locking, and improvements to the method, may still find usefulness. These would include, for example, low-power laser systems and gain media that do not possess a useful Kerr effect.

In this paper we describe a new approach to active mode locking that overcomes some of the traditional barriers to shorter pulse formation by combining multiple (in this case, two) mode lockers within the cavity. It provides a method for reducing pulse width while simultaneously increasing peak power.

Consider the generic actively mode-locked laser shown in Fig. 1. We can drive the arbitrary mode locker with one of the following two methods: 1) a cw signal or 2) short pulses at the laser's fundamental repetition rate. Method 1 will generate short pulses if we use a high frequency, but this leads to a high repetition rate and lower peak powers. Method 2 would be ideal for generating short pulses, but it is extremely difficult to implement because of the required modulator bandwidth.

Many years ago Siegman suggested placing multiple mode lockers in the laser cavity,[5,6] each driven at successively higher harmonics of the fundamental repetition rate. With superposition, these mode lockers represent terms in a Fourier series that, in the limit of large numbers, approximate a delta function. This leads to a very short gain window and hence short pulses. Our technique accomplishes some of the advantages of this approach using two mode lockers. We run one amplitude-modulated (AM) mode locker at the fundamental repetition rate and a frequency-modulated (FM) mode locker at the 22nd harmonic.[7] As an historical note, Kinsel used a single FM mode locker driven simultaneously at the fundamental and the second harmonic of the cavity repetition rate.[8] This is equivalent to two separate modulators, but his goal was to improve the stability of the FM mode-locking process rather than obtaining shorter pulses. Nonetheless, it shares a common principle with our approach. That is, the superposition of harmonics in a Fourier series can enhance an effect at the fundamental while suppressing unwanted effects at higher harmonics.

2. Active AM and FM Mode Lockers

To understand how these mode lockers work together in the same cavity, consider how a single-mode locker

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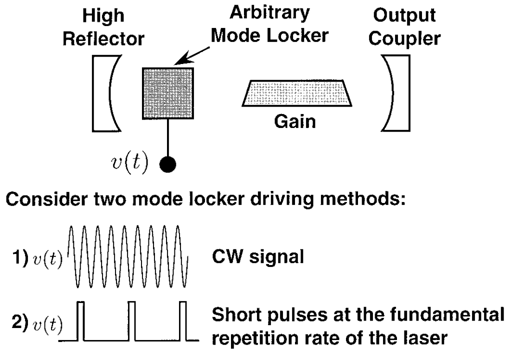

Fig. 1. Generic actively mode-locked laser. The mode locker is driven by the signal $v(t)$ with the goal of producing short pulses. $v(t)$ can be 1) a high-frequency cw signal or 2) a waveform consisting of short pulses at the fundamental laser repetition rate.

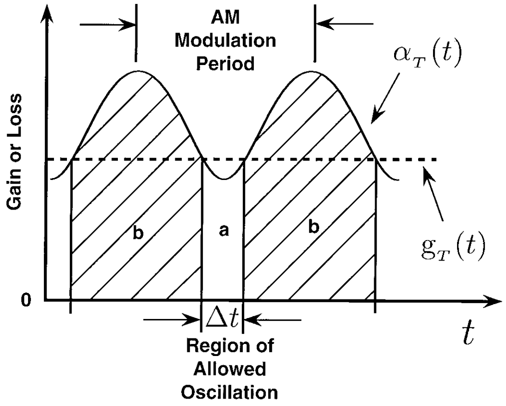

Fig. 2. Integrated round-trip amplitude gain $g_T(t)$ and the integrated round-trip amplitude loss $\alpha_T(t)$ as a function of time for an AM mode-locked laser. When the cavity loss is greater than the gain (region *b*), the cavity will not oscillate. But when the loss drops below the gain (region *a*) the laser can oscillate. This is the region of allowed oscillation and has a width Δt.

influences laser operation. From standard theory,[6] the pulse width is proportional to two parameters, the modulation index and the modulation frequency. This can be written as

$$\tau_p \propto \left(\frac{1}{\Delta_m}\right)^{1/4}\left(\frac{1}{f_m}\right)^{1/2}, \tag{1}$$

where Δ_m is the modulation index and f_m is the modulation frequency. The mode locker drive power P_m varies the modulation index as

$$\Delta_m \propto \begin{cases} P_m, & \text{acousto-optic AM mode lockers} \\ {P_m}^{1/2}, & \text{electro-optic FM mode lockers.} \end{cases} \tag{2}$$

For an AM mode locker, the dependence is linear, and for a FM mode locker it depends on the square root of the drive power. Therefore the pulse width depends weakly on the mode locker drive power (${P_m}^{-1/4}$ for AM or ${P_m}^{-1/8}$ for FM), but more strongly on the drive frequency (${f_m}^{-1/2}$). If we want to shorten the pulse width, we win fastest by increasing the modulation frequency.

Generally for a laser with a single-mode locker, the peak optical power P_{pk} will depend on the modulation frequency,

$$P_{\text{pk}} \approx \frac{P_{\text{avg}}}{\tau_p f_m} \propto \frac{P_{\text{avg}}{\Delta_m}^{1/4}}{{f_m}^{1/2}}, \tag{3}$$

where P_{avg} is the average optical power. For constant modulation index and average power, the peak power decreases as the pulse is shortened because of the increase in repetition frequency. However, this problem can be alleviated if we include two or more mode lockers within the same cavity which forces the repetition rate to be that of the lowest-frequency mode locker.

There are several ways of combining multiple mode lockers. In the case of two mode lockers, either one being an amplitude or a frequency modulator, there are four possible combinations. AM mode lockers are generally acousto-optic and are limited to modulation frequencies less than 1 GHz, making them difficult to use at high harmonic frequencies. However, they have stable mode-locking properties, and, because they modulate cavity loss, AM mode lockers can be used to fix the maximum repetition frequency. Alternatively, electro-optic FM mode lockers are adapted easily to microwave frequencies,[9] but they do not have the stability of their AM counterparts.[8] Also, because they do not affect the gain or loss of the cavity, FM mode lockers are not well suited for preserving the repetition rate in a multiply mode-locked cavity. For these reasons we chose to use an AM mode locker at the fundamental and a FM mode locker at a high harmonic.

To see the effect of using the two mode lockers, let us first look at the time domain representation of AM mode locking. Figure 2 shows the gain curve $g_T(t)$ and the periodic loss curve $\alpha_T(t)$ for a generic AM mode-locked laser. $g_T(t)$ is the integrated round-trip amplitude gain, and it is nearly constant for media with long upperstate lifetimes. $\alpha_T(t)$ is the total round-trip amplitude loss and it oscillates at the amplitude modulation frequency. During mode locking, pulses only exist in the temporal region where the loss is less than the gain (region *a*). We call this the region of allowed oscillation and it has a width Δt.

Next we add a FM mode locker to the cavity and consider its effects within the region *a* (Fig. 3). In conventional FM mode locking, pulses form at a cusp of the FM signal.[6] With the combination of FM and AM mode lockers we have the additional constraint that pulses will form only at FM cusps within the region of allowed oscillation established by the AM mode locker. This is the key to keeping the laser running at the fundamental repetition rate. By running the FM mode locker at a higher harmonic, we obtain the pulse shortening advantages of harmonic mode locking while the AM mode locker preserves a periodic gain window at the fundamental repetition rate. This works, of course, provided that only a single FM cusp resides within region *a*. If the FM

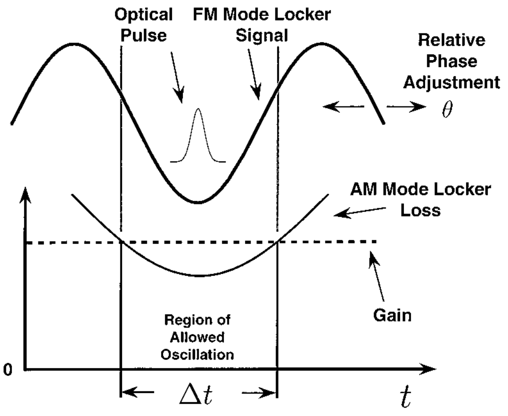

Fig. 3. Expanded view of region *a* (Fig. 2) with the addition of frequency modulation. The relative phase between the AM and FM signals is defined as θ. An optical pulse will reside within the cusp of the FM mode locker signal as long as it is also within the region of allowed oscillation.

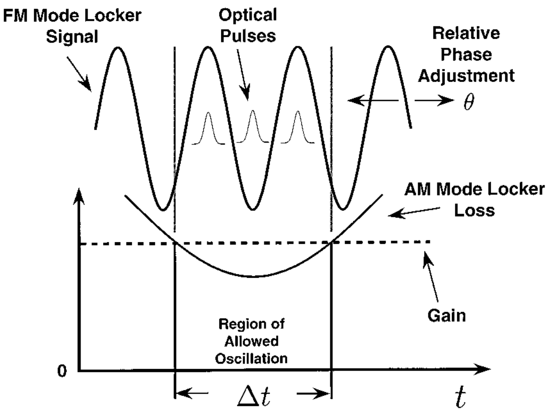

Fig. 4. When the frequency modulation frequency is too large, multiple optical pulses may form within each oscillation region. This results in multiple pulses and unstable laser operation.

signal frequency were, say, two or three times greater than that shown in Fig. 3, two or more cusps could occupy the oscillation region simultaneously (Fig. 4). A pulse could then form under each cusp creating multiple pulses within the oscillation window.[8,10] We, in fact, observed double pulsing and unstable operation when the laser was run in this mode.

When designing a multiply mode-locked laser we want to choose our operating parameters such that we will avoid unstable operation and maximize the laser's performance. A basic approach to determining the optimum parameters of the laser follows.

As we have discussed, the pulse will reside within a region where the gain exceeds the loss, $g_T \geq \alpha_T(t)$. We can also write this as the product of three transmission terms:

$$t_{\mathrm{AM}} t_0 t_g > 1. \tag{4}$$

These terms are defined as

$$\begin{aligned} t_{\mathrm{AM}} &= \exp[-\Delta_{\mathrm{AM}}(1 - \cos \omega_{\mathrm{AM}} t)], \\ t_0 &= \exp(-\alpha_0), \\ t_g &= \exp(g_T), \end{aligned} \tag{5}$$

where t_{AM} is the transmission through the AM mode locker, Δ_{AM} is the peak modulation index (single pass for a traveling-wave cavity and double pass for standing wave), ω_{AM} is the angular modulation frequency, t_0 is the fixed round-trip cavity loss, and t_g is the integrated round-trip amplitude gain. We also conclude that $\alpha_T(t) = \alpha_0 + \Delta_{\mathrm{AM}}(1 - \cos \omega_{\mathrm{AM}} t)$.

When the condition $t_{\mathrm{AM}} t_0 t_g > 1$ is satisfied, the time-varying loss term $\Delta_{\mathrm{AM}}(1 - \cos \omega_{\mathrm{AM}} t) \leq g_T - \alpha_0$. The phase angle $\Delta\theta$ over which this condition is satisfied, is easily found to be

$$\Delta\theta \equiv \omega_{\mathrm{AM}} \Delta t = 2 \cos^{-1}\left(1 + \frac{\alpha_0 - g_T}{\Delta_{\mathrm{AM}}}\right) \tag{6}$$

and corresponds to region *a* in Fig. 2. This phase angle is the oscillation window size in radians with respect to the amplitude modulation frequency. To ensure that only one FM cusp resides within this window, the FM period must then satisfy $T_{\mathrm{FM}} > 2\Delta t$, or equivalently,

$$f_{\mathrm{FM}} < \frac{\omega_{\mathrm{AM}}}{2\Delta\theta}, \tag{7}$$

where $f_{\mathrm{FM}} = 1/T_{\mathrm{FM}}$ is the frequency modulation frequency.

3. Experimental Setup and Results

We demonstrated simultaneous AM and harmonic FM mode locking by modifying a Coherent Antares AM mode-locked Nd:YAG laser. As a standard AM mode-locked laser, it produces 75-ps pulses at an 80-MHz repetition rate with 24 W of average optical power. The FM mode locker used was a microwave resonant $\mathrm{LiNbO_3}$ electro-optic phase modulator[11] operated at 1.76 GHz with a single-pass modulation efficiency of 0.59 rad/(W)$^{1/2}$ (Ref. 12). The crystal size was 4 mm × 4 mm × 40 mm and antireflection coated at 1.06 μm.

To provide phase locking between the AM and FM mode lockers, all RF signals were derived from a common 10-MHz low-noise crystal oscillator. The FM mode locker was placed in the cavity with the center of the crystal 14.8 cm from the output coupler, and the overall cavity length was shortened to compensate for the added delay introduced by the 40 mm of $\mathrm{LiNbO_3}$. When the FM mode locker was in place with no drive signal applied, the average optical power dropped from 24 to 6 W. We believe that this was caused by photorefractive effects within the $\mathrm{LiNbO_3}$ refracting energy out of the $\mathrm{TEM_{00}}$ mode.[11] Also, the laser was somewhat noisy, tending to self *Q* switch occasionally. We attribute the instability to intensity-dependent waveguiding within the $\mathrm{LiNbO_3}$. A mode locker designed for high-intensity intracavity use would avoid this problem.

Simultaneous AM and harmonic FM operation was studied under constant AM mode-locking conditions.

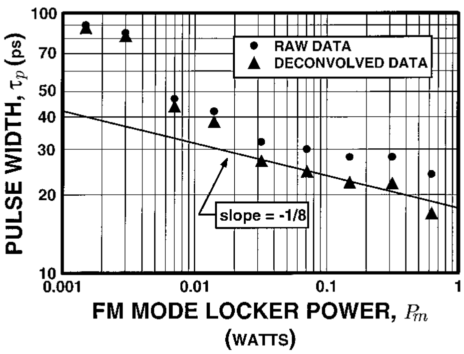

Fig. 5. Optical pulse width versus FM mode locker drive power ($\eta = 0.59$ rad/(W)$^{1/2}$ single pass). Deconvolution was applied to the data because of a 17-ps photodiode impulse response.

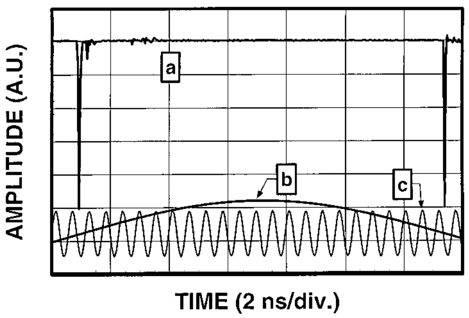

Fig. 6. Verification of fundamental pulse repetition rate. a, Mode-locked pulse train (12.5-ns separation/80 MHz). b, 40-MHz AM mode locker drive signal. c, 1.76-GHz FM mode locker drive signal. The signals are displayed with arbitrary relative phases.

Figure 5 shows the measured pulse width as a function of RF power to the FM mode locker. Also shown is a trend line indicating the $-1/8$ slope dependence (on a log-log scale) predicted by theory [see Eq. (1) and note that $\Delta_m \propto (P_m)^{1/2}$]. At very low RF power (<20 mW) the AM mode locking dominates, but with only a few tens of milliwatts (0.1 rad) the FM mode locker begins to significantly shorten the pulse width and dominate the mode-locking process. When we increased the modulator power to 630 mW (0.47 rad) the pulse width was reduced to 17 ps. The best pulses observed had a peak power of 6.25 kW (16-ps pulse width, 8 W average optical power). This is a 56% increase over AM mode locking alone (~4-kW peak).

Figure 5 shows that our data are consistent with the $-1/8$ trend line for FM mode locker powers above approximately 30 mW. Below this region, the pulse width decreases much faster with increasing mode locker power. We believe this is because two pulse shortening effects occur simultaneously. One is simply an increase in peak modulation depth that is due to the increase in mode locker power, and the other is an increase in effective modulation frequency. Both AM and FM mode locking depend on the modulation frequency according to $\tau_p \propto f_m^{-1/2}$. When the transition is made from purely AM to FM-dominated mode locking, the modulation frequency changes from 80 MHz to 1.76 GHz, which contributes to the rapid reduction in pulse width in the transition region. After this transition, the data follow the $-1/8$ trend line for increasing mode locker power.

The time–bandwidth product ($\Delta\nu\Delta\tau$) for the shortest pulses was 0.54 ± 0.03 or 1.2 times the transform limit for a Gaussian pulse. Although we were not able to verify whether or not the chirp was linear in frequency, FM mode-locking theory predicts chirped output pulses.[6] Also, for pure FM mode locking, the time–bandwidth product should be $\sqrt{2}$ times the transform limit or 0.626. Because we are simultaneously AM and FM mode locking the laser, a time–bandwidth product that falls between the pure FM and AM limits is reasonable. However, further theoretical and experimental investigation of this relationship is warranted.

Figure 6 verifies preservation of the fundamental 80-MHz repetition rate when both mode lockers were operating. The optical pulses (a) as measured with a fast photodiode are shown with (b) the 40-MHz AM and (c) the 1.76-GHz FM mode locker drive signals. We can see that the pulses occur every half-cycle of the AM drive signal (characteristic of acousto-optic mode locking) and every 22nd cycle of the FM drive signal. When the AM mode locker was turned off, the repetition rate increased to 1.76 GHz and the peak pulse power dropped significantly, as expected.

When both mode lockers are operating, it is possible for one to scan the optical pulse across the oscillation window by adjusting the relative phase delay θ between the FM and the AM mode locker drive signals. Time records of the output pulses as a function of θ are shown in Fig. 7. The relative time delay axis for the pulses is referenced to the center of the gain window, and zero degrees relative phase delay places one cusp of the FM modulator approximately at the center of this window.

As the phase delay of the FM drive signal is increased, the FM cusp moves later within the AM gain

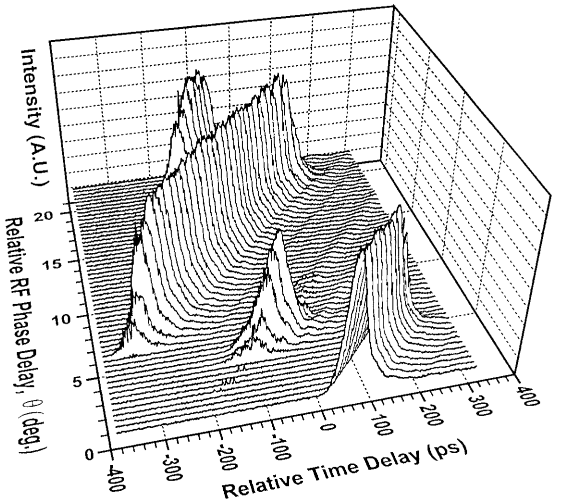

Fig. 7. Variation of mode-locked pulses as the FM signal is delayed with respect to the AM drive signal. This effectively scans the FM signal through the oscillation window shown in Fig. 3. The relative phase delay is in terms of the AM period (i.e., $\theta = \omega_{AM}t$ where t is the rf time delay).

window and the optical pulse follows (i.e., increasing time delay). The peak power of the pulse steadily decreases as it nears the edge of the gain window. When the FM cusp is nearly at the limit of the gain window ($\sim$250 ps), a new pulse appears close to the center. Because the FM period is 568 ps, this corresponds to a new pulse forming under a cusp of opposite curvature. These pulses were found to be less stable, had less peak power, and also appeared within a smaller gain window. When the delay was further increased, this unstable pulse was eliminated, and a new stable pulse emerged at the other end of the gain window beneath a cusp of the original curvature. We speculate that the correspondence between the sign of the FM phase curvature and the mode-locking quality is related to nonuniform axial mode pulling by the closely spaced Nd:YAG laser transition pair at 1.064 μm.[10] Figure 7 also shows that, for certain phase delays (e.g., $\theta \approx 7°$), two pulses can exist simultaneously in one AM cycle. The condition in Eq. (7) was evidently violated by a small amount. However, we verified that these multiple pulses can be eliminated when the gain is lowered (lamp current), thereby decreasing $\Delta\theta$.

4. Conclusion

Simultaneous AM mode locking at the fundamental and harmonic FM mode locking has been shown to be effective at decreasing the pulse width and increasing the peak power of the actively mode-locked Nd:YAG laser. We have demonstrated that the pulse shortening follows the classical theory of Kuizenga and Siegman. With as little as 1 W of microwave power driving the FM modulator, we were able to achieve a fourfold reduction in pulse width. Although the average power output of our laser dropped dramatically, we attribute this loss to material effects and not to the fundamental characteristics of this mode-locking technique. Consequently, with a suitable electro-optic material we should be able to preserve the average laser power and thereby gain a large increase in peak power while maintaining the fundamental repetition rate. This approach as well as other combinations of mode-locking drive frequencies should be applicable to enhancing the performance of other laser systems as well.

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